Instruction for exercise 7

Title: *Experimental verification of Bernoulli's equation*

Theoretical topics

1. Explain Bernoulli's law.
2. Give examples of ideal and non-ideal fluids. Are non-ideal fluids described by the Bernuli equation?
3. Flow of a viscous vs. an ideal fluid.
4. Continuity equation.
5. Derivation of Bernoulli's equation.

Bernoulli's equation is a fundamental relation of fluid mechanics for steady, inviscid, incompressible flow:

$$(1) \quad p + \frac{\rho v^2}{2} + \rho g h = const.$$

where: p is the pressure, ρ is the density of the liquid, v is the velocity of the liquid, h is the height of the liquid in the reference frame.

It links static pressure, dynamic effects due to velocity, and hydrostatic effects due to elevation. In a flowing fluid, dynamic pressure - ($\rho v^2/2$) accompanies static pressure. Flow in a jet can be driven either by a pressure difference or by an elevation difference between its ends.

Measurement procedure

1) Bleeding (de-airing) the system

- Ensure the inlet and outlet valves are closed.
- Open the outlet valve by ½ turn.
- Open the inlet valve by ¼ turn.
- Open the bleed valves on the manometer and on the Pitot tube (grey valves).
- Switch on the pump.
- Slowly open the inlet valve until the manometer fills (water flows through the flow-control valve).
- Wait until air bubbles disappear from the system.
- Open the vent valve on the manometer (white valve).
- Slowly open the outlet valve until the water levels in the manometer tubes begin to drop.

- Close the vent valve.

2) Retract the Pitot tube from the tested cross-section of the Venturi tube.

3) Level the water columns in the manometer.

4) Record the h_{stat} values from the manometer tubes.

5) Record the h_{stag} values for each manometer tube by repositioning the Pitot tube.

6) Retract the Pitot tube beyond the tested cross-section of the Venturi tube (do not remove it completely from the tube).

7) Use a stopwatch to measure the flow time of the liquid.

8) Perform four measurement series. For each series, change the flow rate using the valves and repeat steps 3–7.

Processing of results

1. Compute the volumetric flow rate $\dot{V}$ using equation:

$$(2) \quad \dot{V} = \frac{V}{t} \left[\frac{m^3}{s}\right],$$

   where: V is the volume of liquid, t is the flow time.

2. Compute the mean velocity of water at each point along the Venturi using the continuity relation:

$$(3) \quad v = \frac{\dot{V}}{A}\left[\frac{m}{s}\right],$$

   where: v is the fluid velocity at a given point in the orifice, A is the cross - sectional area of the orifice at a given point.

3. Compute the dynamic pressure at each point from equation:

$$(4) \quad h_{dyn} = h_{stag} - h_{stat}[mmH_2O].$$

4. Compute the water flow velocity at each point from the simplified Bernoulli equation for a horizontal tube:

$$(5)\ p + \frac{\rho v^2}{2} = const.$$

The first term of the equation corresponds to the static pressure, the second to the dynamic pressure. The sum of these pressures allows us to determine the stagnation (total) pressure, which allows us to write the relationship:

$$(6) \quad p_{stag} = p + \frac{\rho v^2}{2}.$$

By performing appropriate transformations we obtain the formula for the velocity of the flowing liquid:

$$(7)\ v = \sqrt{\frac{2(p_{stag} - p_{stat})}{\rho}}.$$

Using the formula:

$$(8)\ p = \rho g h [Pa],$$

the velocity can be determined as:

$$(9)\ v = \sqrt{\frac{2(\rho g h_{stag} - \rho g h_{stat})}{\rho}} = \sqrt{2 g h_{dyn}} \left[\frac{m}{s}\right].$$

5. Calculate the pressure change using the formula:

$$(10) \qquad \Delta h = h_{stag1} - h_{stag6},$$

where h_{stag1} is the pressure from the Pitot tube at point one, h_{stag6} is the pressure from the Pitot tube at point six.

6. Plot a graph of the fluid velocity versus the Venturi point.
7. On a single set of axes plot: h_{stat}, h_{dyn}, and h_{stag} vs. the Venturi point.

References

- Dryński T., Laboratory Exercises in Physics, 6th revised ed., PWN, Warsaw, 1977.
- Szczeniowski Sz., Experimental Physics, vol. 1, 1st ed., PWN, Warsaw, 1972.
- Halliday D., Resnick R., Walker J., Fundamentals of Physics 2, 2nd ed., PWN, Warsaw, 2015.